Lethology
A Theory of Learning and Knowing vis à vis Undeterminables, Undecidables,Unknowables(*)

Heinz ■ Von ■ Foerster

First of all I want to thank the Authorities of the Comprensorio di Primiero, the sponsors and organizers of this Symposium on *Cognition as Education*, who gave us a unique opportunity not only to learn about the many perspectivas of learning and knowing, but also to meet old friends and colleagues from different parts of the globe, and to make new ones here at this magnificent place.

Heinz von Foerster. Profesor de la Universidad de Ilinois. Investigador en Cibernética, Sistémica y Filosofía.

(*) This article is an adaptation of an address given on April 26,1990, in San Martino di Castrozza at the Seminario Internationale «Conoscenza come educazione».

I was very impressed by the courage of the organizers to invite Americans, namely my colleagues and me, to this symposium on education, though it is well known that the United States of America have one of the worst educational systems of the Western world.

Let me read from the May 1989 Edition of the Review Draft by the Curriculum Commission to be submitted to the California State Board of Education (1):

..."In 1983, *A Nation at Risk* declared that American education had become victim to a 'rising tide of mediocrity.' The National Science Board's Commission an Precollege Education in Mathematics, Science and Technology confirmed that the situation in science education was particularly critical and recent studies have placed America's students last among their internacional counterparts in understanding science. In 1988, the National Assessment of Educational Progress of the Educational Testing Service issued *The Science Report Card*, and noted that while the responses in the intervening years since 1983 have resulted in some progress, 'average science proficiency across the grades remains distressingly low'."

Perhaps we have been invited to find out how *not* to set up an educacional system.

Why indeed does American education function so poorly?

To start with, there is great difficulty in setting up an educacional strategy in a pluralistic society with a population of diverse cultural roots from Europe, Africa, Asia, and Latin America.

But many other causes are invoked as well, rightly or wrongly, for explaining the failure of the American school system: too much emphasis on sports; overburdened and underpaid teachers; decisions on how and what to teach left to local school boards; politization of textbooks; and so on and so forth.

May be all of the above contribute to the appalling results, but I think it is the choice of an epistemology, a theory of knowledge, that is counterproductive, even inhibitory, to the cognitive processes of appreciation, fascination, enthusiasm, curiosity, etc., that are prerequisite for learning and understanding.

Let me take as an example again the report to the California State Board of Education I mentioned before. On its almost 200 pages there is not a single sentence that addresses the questions of how do students learn, what takes place in the mind of the learner and what this gigantic machinery we call "schooling" is all about.

Indeed, what is learning?

If this question is asked in an academic context say, in departments of education or psychology there will be many answers. However, if this question is asked in an operational context, there are no answers at all: we have not the slightest idea of what is going on within us when we say we have learned something.

I mean by that that here we are all walking talking and socializing ever since we were about two years of age, although we never took courses in our mother tongue nor in the art of locomotion. There were no curricula regarding these faculties, and we have no idea how we acquired them.

Indeed, what is learning?

If this question is asked in an academic context say, in departments of education or psychology there will be many answers. However, if this question is asked in an operational context, there are no answers at all: we have not the slightest idea of what is going on within us when we say we have learned something.

The denotative school of language acquisition will argue that we understand very well how we learn to speak, namely, by imitating those who point to things and make the appropriate noises. But I learned from Margaret Mead, the anthropologist, who easily picked up colloquial language of the many different tribes she worked with, that this is not so. Once she used this method by pointing to different things hoping to learn how to call them. To her dismay she always got the same answer "chumumula". First she thought they have a very primitive language until she found out that "chumumula" means pointing with one's finger.

The Noam Chomsky school of thought will argue, that we know very well how we learn to speak, namely, by activating a "language organ" that is grown into our body. Following this train of thought I would propose a 'mathematical organ" to explain our mathematical abilities, perhaps with special organelles for addition, subtraction and multiplication.

These and similar notions fall into the category of "explanatory principles", a category which was invented by Gregory Bateson to answer "What is..." -questions from his daughter (2). When she asked: "Daddy, what is an instinct?", he answered: "An instinct, my dear, is an explanatory principle." And when she wondered what does it explain, he said: "Anything - olmost anything at all. Anything you want it to explain." When she protests that it could not explain gravity Bateson retorts; "But that is because nobody wants 'instinct' to explain gravity. If they did, it would explain it. We could simply say that the moon has an instinct whose strength varies inversely with the square of the distance...", whereupon she stops him: "But this is nonsense!" and he: "But it was you who mentioned instinct, not I", etc.

I leave it to you to follow up on these charming "Metalogues" as Bateson called them; my point here is to draw your attention to such stopgaps of inquiry of which there are, besides "language organs", "instincts" many others, e.g., "drives", "mind", "memory" (3), etc., etc. They always come handy when we do not know what is going an.

With the following example, however, I would like to invite you to come with me to the brink of the abyss of our fundamental ignorance, and to stand with me in awe before the vastness of this void.

I take this example from a book by the psychiatrist Oliver Sacks *The Man Who*

Mistook His Wife for a Hat (4). Among the many cases of the astounding functioning of "dysfunctional" minds, the most fascinating for me is his report about a pair of twins he once met in a state hospital, John and Michael, who were variously diagnosed as autistic, psychotic, or severly retarded. Here is his description: "They are... unattractive at first encounter, a sort of Tweedledum and Tweedledee, indistinguishable, mirror images, identical in face, in body movements, in personality, in mind, identical too in their stigmata of brain and tissue damage. They are undersized, with disturbing disproportions in head and hands, high-arched palates, high-arched feet, monotonous squeaky voices, a variety of peculiar tics and mannerisms, and a very high, degenerative myopia, requiring glasses so thick that their eyes seem distorted, giving them the appearance of absurd little professors, peering and pointing, with a misplaced, obsessed, and absurd concentration."

When he first met them, they were already known as having a remarkable "documentary" memory, that enabled them, for instance, to say at once on what day of the week a date far in the past or future would fall.

However, he did not think about them, until he had another encounter of which he writes: "I forgot [them] until a second, spontaneous scene, a magical scene, which I blundered into, completely by chance."

"The second time they were seated in a corner together, with a mysterious, secret smile on their faces, a smile I had never seen before, enjoying the strange pleasure and peace they now seemed to have. I crept up quietly so as not to disturb them. They seemed to be locked in a singular, purely numerical, converse. John would say a number, a six-figure number. Michael would catch the number, nod, smile and seem to savour it. He, in turn, would say another six-figure number, and now it was John who received and appreciated it richly. They looked, at first, like two connoisseurs wine-tasting, sharing rare tastes, rare appreciations. I sat still, unseen by them, mesmerised, bewildered."

"*What* were they doing? What on earth was going on?" Oliver Sacks asked himself. But since he is not only a psychiatrist but also a numbers buff, he could provide at least a clue: while the twins were playing their game with numbers, he wrote them down and looked them up later at home in a book that lists all prime numbers up to nine-figure primes. Prime numbers are those peculiar islands floating in the infinite sea of numbers that do not evenly divide by any number but by themselves or one. To his amazement, his hunch was correct: *all* the six-figure numbers the twins exchanged were primes! This persuaded him to join them the next day, and equipped with his prime number book he presented them with an eight-figure prime: "... They both turned towards me, then suddenly became still, with a look of intense concentration and perhaps wonder on their faces. There was a long pause - the longest I had ever known them to make, it must have lasted a half-minute or more - and then suddenly, simultaneously, they both broke into smiles."

Now all three were playing the game, with the prime numbers getting larger and larger, until the twins were coming up with numbers much larger than those in the book. When they moved on, swapping twenty-figure numbers, Oliver Sacks could only sit in amazement, watching an unfathomable prime-numbers-ping-pong game, and contemplating unfathomability.

From this example I learned at least one thing, namely, that we generally do not appreciate our own miraculous faculties when they work. We are, however, surprised when they don't work in the usual way and manifest themselves in other forms. Since there is not the smallest handle in sight with which to grasp the enigmatic behavior of the twins, I claim we are precisely in the same situation when we wish to grasp our own.

Vis à vis this enigma and vis à vis our ignorance and, paradoxically, vis à vis our sense of knowing, I thought about an epistemology, a theory of knowledge, that is cognizant of the vastness of our ignorance, a tip-of-the-iceberg epistemology that is aware of its floating state of affairs.

May be one could call this a development for a calculus of un-knowables, or a theory of un-knowledge, but I was unhappy with the negative connotation implied by the prefix "un-", and I looked for a word that would refer to the absence of a faculty in a positive sense, as blindness is "un-seeing" or deafness is "un-hearing".

Neither in Greek nor in Latin I could find what I was looking for, and I was on the verge of giving up my search, when I remembered that "truth", believed by many today to have a positive sense, had a negative connotation in ancient Greece: "Aletheia", or "that which is *not* obscured", with the prefix "a" for "not", and "letheia" from "lanthano" to "hide" to "obscure". It is the river Lethe, you may recall, one crosses to enter Elysium and all memories vanish, while crossing the river Acheron all memories are reinforced before entering Hades, so that they haunt you *ad infinitum*.

Thus Lethe offeres itself naturally for naming a calculus of unknowables "Lethology", and I am going to use this calculus as a rigorous platform for discussing the problems I mentioned before.

I see within unknowables two components: undeterminables and undecidables; thus I shall address these in the following two points:

1. How to deal with systems that are in principle undeterminable; and
2. How to answer questions that are in principle undecidable.

Thus Lethe offeres itself naturally for naming a calculus of unknowables "Lethology", and I am going to use this calculus as a rigorous platform for discussing the problems I mentioned.

1. UNDETERMINABLES

"Causality determines the flow of events in the universe" is one of the central beliefs in our Western culture. It is the belief that if we were to understand the Laws of Nature we would understand the world: our quest, therefore, is to determine these Laws.

While skeptics argued for over two and a half millenia against this belief, I would like to report about some other arguments in the same direction that were developed over only the last fifty years by logicians and mathematicians who studied the fundamental principles, functions, and operations of systems in general, because the results of these studies have a direct bearing on the central themes of our symposium, namely, cognition, education, and learning.

Although I shall discuss theoretical aspects of these notions, there is no need to go through logico-mathematical acrobatics that would be hard to follow by the uninitiated. Thanks to an elegant intellectual twist invented by the British mathematician Alan Turing (5), we can leave all the cumbersome derivations, deductions, inferences, etc., to a (conceptual) "machine", and can comfortably sit back and watch the machine grinding out answers for our illumination and contemplation.

A "machine" in this context is a set of rules by which some state of affairs are transformed into some other state of affairs. For our purpose it is sufficient to distinguish only two kinds of such machines: one, usually referred to as "trivial machine", has only one fixed rule that operates without change on the various state of affairs; the other one, the "non-trivial machine", having rules which, however, change the *rules* that operate an the state of affairs: a machine within a machine, a "second order machine", so to say.

To make these notions more tangible let me first construct, or synthesize, a typical trivial machine whose "state of affairs" consists of only the first four letters of the alphabet A, B, C, D, and whose "rule of transformation" is to associat (anagrammatically) each state (letter) with one in the opposite sequence D, C, B, A. If one presents this "anagrammor' with, say, a "B", it will respond wih a "C", and - *mutatis mutandis*- will do so in perpetuity.

As you will see at once, the trivial machine is one of the central pillars of Western thought (6). Take, for instance, the mini-universe of the four letters of before, together with the anagrammatic rule of transformation as metaphor for a universe with four states of affairs and the transformation rule as its Law of Nature. Then with A, B, C, D, as causes and D, C, B, A as effects, causality is now determining the flow of events in this Universe; or take a transformation rule as the property of an organism, then certain stimuli will elicit the appropriate responses; or take the character of a person as a transformation rule, then his or her motives will entail their corresponding actions; or look at computer science where the transformation rule is, of course, the program computing from its inputs the appropriate outputs.

The underlying triadic structure of all these examples is that of logical syllogisms with their two premisses and their inescapable conclusion. This structure is also embedded in our language through the words "because", "in order to", and "for", usually with an unspoken reference to an immutable rule. However, remember when I proposed in my earlier example a partircular transformation rule, a Law of Nature, it was *my* choice. In other words, I was playing God for this Universe, for I could have chosen other Laws, for instante replacing each letter with its follower (and D with A); or pulling the four resulting letters out of a hat, letting chance determine the Laws of Nature (in contrast to other beliefs (7)). etc., etc.

It is important to see the considerable freedom we have in synthesizing these machines. But it is also important to see that if we do not know their workings we can through examination identify the operations of such machines. One has simply to go through all available states (A, B, C, D) and pair them with their responses (say, D, A, B, C): then the pairs AD, BA, CB, DC, identify this anagrammatic machine, for they *are* the "machine".

In other words, trivial machines are not only determined through their synthesis, they are also determinable through analysis. Morover, since their operating rules remain unchanged, i.e., they are historically independent, thay are also predictable!

lt was apparently this insight that prompted Laplace almost 200 years ago to make his paradigmatic statement (8) that if for a superhuman intelligence the present conditions of all particles in the universe would be known "... nothing would be uncertain and the future and the past would be present to his eyes." Today Laplace would rejoyce: "The Universe: a trivial machine!"

I am sorry to say, mon cher Pierre Simon Marquis de Laplace, you rejoyced too early.

This is the appropriate moment to turn to non-trivial machines. As I described them before, they are changing their operating rules according to a "second order" rule, a "program". Such a program may be seen as operating on "internal states of affairs", the operating rules of before, namely, those that operate on the "external states of affairs". To stay with the earlier example of the "anagrammor", we are now changing the anagrams according to the program that specifies a particular non-trivial machine.

If we contemplate changing one anagram into another, we have to know the anagrams at our disposal. Referring again to our 4-letter universe (A,B,CD), then, depending on whether we allow different letters (say, A, B) to be translated into like letters (say, A-C; B-C) or not, we have with 4-Letter anagrams for the two cases (see Appendix (Apx)):

$$N_{like} = 256 \text{ anagrams}$$
$$N_{diff} = 24 \text{ anagrams}$$

For the purpose of constructing now a non-trivial machine, I have listed (Apx) and numbered all 24 anagrams possible with all four letters different; and for the purpose of demonstrating the workings of such a machine I picked for the "internal states of affairs" the four anagrams #10, 17, 19, 24, where #24 you may recognize as the one used in the example for the trivial machine:

10	17	19	24
A	C	D	D
C	D	A	C
D	A	B	B
A	B	C	A

In fact, whenever our to be constructed machine stays in any one of these internal states, it acts as a trivial machine according to that anagrammatic rule.

Now we are in a position to select the program that will run our machine; this program determines the letters X′ and the rules (anagrams) R′ that follow after the machine operated on letter X and under rule R. For this demonstration I chose the following program of operations:

TRANSITION TABLE

R

X		R = 10		R = 17		R = 19		R = 24	
		X′	R′	X′	R′	X′	R′	X′	R′
	B	B	10	C	17	D	19	D	24
	B	C	17	D	19	A	24	C	10
	C	D	19	A	24	B	10	B	17
	D	A	24	B	10	C	17	A	19

Example: Assume the machine is in a state in which it computes for A, B, C, D the anagram #10 (B, C, D, A), and is presented with the letter B (X=B); of course, it will produce C (X′=C), but at the same time it will change the anagrammatic rule from #10 (R=10) to #17 (R′=17). Hence, if given B again, we have to look under R=17 and find the response to be D instead of C as before! Since the machine has moved now into #19, B again produces A, etc., etc. Below is for a repeated sequence of A, B, C, D (first line) the sequence of responses (second line):

A, B, C, D, A, B, C, D, ...
B, C, A, A, D, A, B, B, ...

I hope these examples are sufficient to demonstrate the fundamental difference between these machines and their trivial sisters. However, when the Transition Table of above is given, the determination of any sequence is now trivial. Why then calling these machines non-trivial? This will become obvious when we do not know the program or transformation rules, and have to identify them through experimentation.

Before planning such an experimental procedure it would be wise to estimate the effort that has to go into, solving the identification problem. This effort depends, of course, upon our knowledge of the system. Let us assume we know that the program works (as in our example) with exactly 4 anagrammatic rules and the alphabet consists of precisely 4 letters; then the number of different machines, one of which is the one we want to identify, is precisely (Apx):

$$N_4 = 4{,}294{,}967{,}296.$$

This looks like a large number, but with large number-crunching computers that may test one million of our possible machines in one second, it takes perhaps not more than 1 hour and 15 minutes to have our machine identified. But let us assume now that we do not know that only 4 anagrammatic rules (i.e., Laws of Nature) are operative, but we do know that this universe has the property that two different "causes" will never result in like "effects", then the number of possible universes, of which ours is only one, is (Apx):

$$N_{24} = 6.3 \times 10^{57}.$$

Since the large computer of before can test only about thirty trillion (i.e. $30x10^{12}$) machines per year, and the universe we are living in is at most only 20 billion (i.e., $20xlO^{9}$) years old, it is much too young to have tested only a fraction of the possibilities large enough to be mentioned.

But our ignoranse may run deeper. Since distinguishing difference in causes and difference in effects is a question of cognitive skills, we cannot be certain that the canon "different cause/different effect" is valid for the universe under investigation. Hence we have to be prepared to look for the one sample out of (Apx)

$$N_{256} = 5 \times 10^{616}.$$

This is a number with 616 zeros following 5. Clearly, the identification problem of non-trivial machines is non-trivial or, as it is put in the language of computer scientists, it is "transcomputational". Translated back into common language: it can't be done!.

Optimists, nevertheless, may argue that sooner or later we will have the theoretical or technical means available to tackle this problem. Unfortunately, however, this hope is unfounded. It can be shown (9) that there are machine configurations whose identity cannot, in principle, be established by a finite sequence of experiments: the machine identification problem is in principle unsolvable!

While non-trivial machines can be synthetically determined, they are analytically undeterminable, history dependent, and unpredictable.

It takes a long time for this insight to sink in, for it contradicts all the intuitive notions we have of Nature's magnificent order, of the reliability of our friends and of a coherent sense of ourselves. Shall we doubt the non-triviality of all this?

When I ask my friends whether or not they consider themselves to be trivial or non-trivial "machines", they unequivocally opt for non-triviality, although when asked of their opinion about others, the answers are mixed. This should not surprise, because in comparison to the fickle, unpredictable, and unanalyzable non-trivial machine, the trivial machine with its reliability and predictability appears to be a gift from Paradise. We pay considerable sums of money for garantees that the machines we buy are not only trivial when we buy them but maintain their triviality for a long period of time. When one morning our car refuses to start, its history dependent, non-trivial, true nature comes to the fore, and we have to call a professional trivialisateur who, with his tools re-establishes the car's apparent trivality.

It is clear that we as children of our culture are infatuated with trivial systems and whenever things do not go the way we think they should go we will trivialize them: then they become predictable.

I have discussed this point at length, because at some uneasy moments I sense that in the absence of an understanding of how to deal with one of the most non-trivial, inventive surprising, unpredictable creatures I know of, our children, some educational systems confuse learning with trivialization. In learning, the number of internal states grows and the semantic relational structure (the "program") becomes richer. Trivialization, on the other hand, means amputation of internal states, blocking the evolution of independent thought and rewarding prescribed, hence predictable, behavior: "6" *is* the answer to the question "What is 2x3?"; unacceptable is: "an even number", "3x2", "my age", and others (10).

This begs the question of the meaning of "tests". Are tests designed to establish the workings of another mind, the mind of the student? Then tests try to establish the impossible, for as we know now, the mind of a non-trivial student is analytically indeterminable. Are tests designed to establish the degree of success an educational system had in trivializing its students? Then the results do not reflect upon the malleability of students but upon the educational system and the tests it designs. That is:

tests test tests,
(and not those supposedly tested)

examinations examine the examiner, not the examined. This becomes evident when we see students study exams in order to pass exams, which is not the same as to study the subject matter to know the subject matter. But then the question arises of how do we know what they know of the subject matter? Indeed, how do we know? Or, in this context, how do we know vis à vis unknowables? Or, using the concept of non-trivial systems, what is it that we may know about them, if we cannot know about their workings?

This begs the question of the meaning of "tests". Are tests designed to establish the workings of another mind, the mind of the student? Then tests try to establish the impossible, for as we know now, the mind of a non-trivial student is analytically indeterminable. Are tests designed to establish the degree of success an educational system had in trivializing its students? Then the results do not reflect upon the malleability of students but upon the educational system and the tests it designs. That is: tests test tests, (and not those supposedly tested).

25 years ago this question was either not asked, rejected when asked, or one thought about developing methods of trivialization. However, within these years the pursuit of two ideas changed fundamentally the approach to this question, One of these ideas evolved in mathematics where the notion of non-triviality was translated into non-linearity, which brought forth a colorful bouquet of surprises and amazements under the names of non-linear dynamics and chaos theory; the other one grew within systems and computer science, where the cybernetic notion of circularity and closure brought forth a wealth of new insights and

perspectives.

Let me first address the notions of circularity and closure, for these are notions that were avoided by orthodox science. What is meant by circularity is that the outcome of the operation of a system initiates the next operation of that system: the system and its operations are a "closed system". This is to allow that an experimenter considers her- or himself as part of the experiment; or that a family therapist perceives of him or herself as a partner of the family; or that a teacher sees her- or himself as participant in the learning/ teaching process in the classroom, etc., etc.

To see the unorthodoxy of closure, i.e., including the actor into the actor's universe, one should remember Laplace, who did not think of himself being a part of his universe, otherwise he would not have declared it to be a trivial machine, or think of "objectivity": it demands that the properties of the observer shall not enter into the description of his observations. I ask how can this be done? Without him there would be no description nor any observation.

I shall now report on the fast developing field of non-linear dynamics, where circularity and closure are essential ingredients. There are three results that are here of fundamental significance. The one is that if one lets a closed system operate recursively on its outcome, it will sooner or later, converge to a stable behavior. Because of the short history of this development, these dynamic equilibrio are by different people given different names: "fix points", attractors", "strange attractors", and "eigen-behaviors" (11). I shall demonstrate these dynamic stabilities in a moment, after I have accounted for the other fascinating results of these studies. One of them is that the initial condition of some systems determine their behavior in a crucial way: they may under one condition assume one, two, or more different forms of eigen-behaviors or under another condition, will go on and on without ever going into any form of stability: they become chaotic! One of the surprising observations here is that frequently the two initial states which separate the system's behavior into convergence to stability or into divergence to chaos can be infinitesimally apart: even systems whose rules of operation are known may be unpredictable!

An example shall clarify these points. Let us operate recursively the non-trivial machine of before offering it the letter "A", and asuming it to be in the initial state of computing anagram #10 (A/10). From the Transition Table (page 8) we see the response to be B/10. B/10 now recursively re-enters the machine to produce C/17 and, recursively A/24, and so on and so forth. Below is the sequence of the first 17 events (top row letter/bottom row number of state):

A, B, C, A, D, A, D, C,
A,D, A, D, C, A, D, A, D, ...
10 10 17 24 24 19 19 17 24 24 19 19 17 24 24 19 19, ...

As can be seen, after a transient period of only two steps (A, B), the machine converges to a dynamic stability that manifests itself in producing periodically the sequence CADAD, the eigen-behavior of this machine under the chosen initial condition. Let us try another experiment by starting with a condition that did not appear in the previous run, say C/24:

C,	B,	D,	C,	A,	D,	A,	D,
24	17	19	17	24	24	19	19

There, after 3 transient steps (C, B, D) the system assumes a stable dynamics, the eigen-behavior of before: CADAD, CADAD appears, so to say, to be the manifestation of this machine's inner workings which, for those who do not know it, will remain an unknowable for ever.

I leave it to the curious among you to find out whether the machine so assembled is capable of other dynamic stabilities, or whether CADAD is the only thing it can say of itself.

I shall go on to generalize these observations in three steps. The first is to give without proof the essence of a theorem concerning these machines. It says that an arbitrarily large closed network of recursively interacting non-trivial machines can be treated as a single non-trivial machine operating on itself, as, for instance, the machine in our example. This insight entails the second point, namely, that the dynamics between all interacting participants in such a network will converge to a stable dynamics, to the eigen-behavior of this network. The third step, or should I call it a leap, now follows: Let the interacting participants be the participants in a social network, then their eigen-behavior manifests itself in the language spoken, the objects named, the customs maintained, the rituals observed. Embedded in this network are the "teachers" and the "students" who, through their dialogue, establish an understanding, not of each, but of *each other* where a subject matter may be the vehicle for this understanding, for learning how to learn.

How this comes to pass is unknowable; but that it comes to pass is because of our doing it together in a recursive dialogue. Here is what Martin Buber has to say (12).

"Contemplate the human with the human, and you will see the dynamic duality, the human essence, together: here is the giving and the receiving, here the agressive and the defensive power, here the quality of searching and responding, always both in one, mutually complementing in alternating action, demonstrating together what it is: to be human. Now you can turn to the single one and you recognize him as human for his potencial of relating; then look at the whole and recognize the human for his richness of relating. We may come closer to answering the question: what is a human?, when we come to understand him as the being in whose dialogic in his mutual present two-getherness, the encounter of the one with the other is realized at all times.

2. UNDECIDABLES

There are among propositions, problems, questions, etc., those that are decidable and those that are in priciple undecidable.

Decidable, for instance, is the question whether 3,536,712 is, without remainder, divisible by 5. The answer is clearly a "No"; however, if we had asked "divisible by 2?", the answer clearly is "Yes". One could, of course invent more difficult questions, very difficult questions, extraordinary difficult questions that may take years to decide, but in the pursuit of answering them we are assured of their decidability because of our choice of the rules how to climb from one node in this crystalline structure of logico-mathematical relations to the next one.

This is why for example mathematicians are, after 250 years, still trying to "proof", that is, to give explicit instructions to the climbers of how to procede, a conjecture that Christian Goldbach wrote in 1742 in a letter to Leonard Euler. Goldbach had the hunch that every even number can be representad by the sum of two primes, as, for instance, 12 = 5+7, or 16 = 13+3, etc., etc.

Indeed every even number tried so far can be decomposed into two primes but this is, of course, not a proof! In other words, the question: "Is Goldbach's conjecture provable?" is not (yet) decided. By inserting the three letters "yet" in the previous sentence decidability is stipulated. But with Kurt Godel's observation in 1931 that within our mathematical system there are undecidable propositions (13), the suspicion arises that Goldbach's conjecture may be one of them (14).

But there is no need to participate in logico-mathematical somersaults to appreciate the appearance of in principle undecidable questions for everyday language and lore is laced with them. Take the question of the origin of our Universe: how did it come about? Clearly, this question is in principle undecidable, for there could not be any witnesses, and if there were, who would believe them. Nevertheless, there are many answers to this question. Some say it was the union of Chaos with Darkness that brought forth all there is; others say it was a singular act of creation some 4000 years ago; others insist that there was no beginning and there is no end because the universe is in a perpetuos dynamic equilibrium an "eigen-universe"; still others argue that the whole thing began 10 or 20 billion years ago with a Big Bang, whose noise can still be heard as a whisper over large radio antennas; I have not yet accounted for the answers Hindus, Arapesht Massaist Nubas, Khmersp Bushmen, etc., etc., would give when asked this question. In other words, tell us how the Universe began, and we tell you who you are.

The distinction between decidable and in principle undecidable questions may be now sufficiently clear, that I can present to you the following thesis (15):

"Only those questions that are in principle undecidable, *we* can decide".

Why?

Simply because decidable questions are already decided by the choice of the framework within which they are asked. The frame work itself may however, have been an answer *we* chose to an in principle undecidable question. This observation sharpens the distinction between these two kinds of question: answers to decidable questions are forced through necessity, while for those to undecidable questions *we* have the freedom to choose. But with this freedom of choice we must assume the responsibility for our choice. This sharpens further the distinction between these questions: procedures for arriving at answers to decidable questions may be faulty, hence, here arises the notion of truth; ethics however is the domain within which we assume responsibility for our decisions: the antonym for necessity is not chance (16), it is freedom, it is choice.

The distinction between decidable and in principle undecidable questions may be now sufficiently clear, that I can present to you the following thesis: "Only those questions that are in principle undecidable, *we* can decide".
Why?
Simply because decidable questions are already decided by the choice of the framework within which they are asked. The frame work itself may however, have been an answer we chose to an in principle undecidable question. This observation sharpens the distinction between these two kinds of question: answers to decidable questions are forced through necessity, while for those to undecidable questions we have the freedom to choose.

How do these considerations affect our perspectives on cognition, on learning, and on "Cognition as Learning"? I think in a crucial way. Here some examples:

Mathematicians dwell in two distinct worlds that are irreconcilably separated by deciding differently the in principle undecidable question "Are the numbers, the formulas, the theorems, the proofs, etc., of mathematics discoveries or are they our inventions?".

Here is a report (18) about the way a citizen of the world of discoveries sees how we know mathematics: "A deity he fondly calls the Supreme Fascist 'has a transfinite book of theorems in which the best proofs are written. And if he is well intentioned, he gives us the book for a moment.' Like a medium at a seance it is said, a good mathematician is one who is especially adept at communicating with this Platonic realm where abstractions and symmetries sit waiting to be discovered by the properly prepared mind.'

And here is the confession of a citizen of the world of inventions (19): "I for my part believe that what a mathematician does is nothing but the derivation of statements with the aid of certain, to be enumerated and in various ways choosable, methods from certain, to be enumerated and in various ways choosable, statements and all what mathematics and logic can say about the mathematician's activity,... is contained in this simple statement of the state of affairs.

Let us consider children growing up in these two different worlds: In the world of discoveries they must learn to repeat what others were by the Supreme Fascist permitted to glance from "The Book"; in the world of inventions they are invited to play a game in which they write the rules, invent their mathematics, from which mathematicians may learn one thing or another (20).

Here another example:

Ever since the French psychologist Alfred Binet invented a century ago a test for intelligence, the belief in the ability to test for intelligence became very popular indeed, surprisingly more so in Brittain and in the United States than in the country of its origin. It is therefore understandable that when electronic computers became more and more sophisticated, the question of whether these "chaps" are intelligent, and how to decide whether they are, was first raised by an Englishman, our friend Alan Turing, the inventor of the non-trivial machine.

The test he proposed to establish whether or not computers can "think" has now become the *credo* for those who believe in Artificial Intelligence, or AI. The test consists of having an "X". which can be a human being or a computer, placed behind a curtain, and having examiners bombarding "X", with questions to find out what is behind that curtain: man or machine? lf they erroneously conclude "X" is a human being, or if they cannot decide and give up, it is said that the computer has tested positive an the Turing Test, that is, this computer is intelligent, this computer can think'. AI is justified (16)!

It is always surprising and amusing to me that it is not plain to everyone that it is not the machine that has passed the test, but that it is the examiners who have failed it by making wrong judgements or by accepting defeat. This surprises me the more, since the problem of "The Other Mind", that is, "Are there other minds besides me?" is, with a few Continental exceptions, a problem confined to the Isle (17), and those philosophers who pursued this problem would not have accepted the Turing Test to decide it.

At this point you may have guessed that I would like you to see "The Other Mind" and related questions as in principle undecidable, hence for *us* to decide and to take the responsibility for our decisions.

lf you have followed me so far, I ask you to stay with me through the next points, though they may hurt first before they take shape. I take the metaphor of seeing the question of "The Other Mind", that is, "Does X have a mind?", to let the answers to the questions "is X incompetent?", "is X a criminal?", "is X insane?", etc., to be seen as being the responsibility of those who decide these questions: the examiners, the jurors and judges, the psychiatrists, etc.

This points to the ontological trap where attention is placed on the *is* in the question "*is* X insane'?", instead of directing the attention to Y who decides (for her or himself) what "is".

Ontology, and objectivity as well, are used as emergency exits for those who wish to obscure their fredom of choice, and by this to escape the responsibilty of their decisions. Here is José Ortega y Gasset's observation (21):

"Man does not have a nature, but a history... Man is no thing, but a drama... His life is something that has to be chosen, made up as he goes along, and a man consists in that choice and invention. Each man is a novelist of himself, and though he may choose between an original writer and a plagiarist, he cannot escape choosing... He is condemned to be free."

Indeed, we are condemned to be free!

Let us rejoice in this freedom by joining the chorus in Beethoven's Ninth Symphony with the new version of Schiller's words where "Freude" (joy) is now sung all over the world as "Freiheit" (freedom) (22):

Freiheit schöner Götterfunken
Tochter aus Elysium
Wir betreten feuertrunken
Himmlische dein Heiligtum.

APPENDIX

(i) The number of ways in which n different objects can be put into n different boxes, called permutations is:

$N_{diff} = n! = 1, 2, 3 \ldots . (n-1), n;$

In our case, n=4:

$N_{diff} = 4! = 1, 2, 3, 4 = 24.$

(ii) The number of ways in which n symbols (say, numbers) can be written in strings of p places (digits) is:

$N_{like} = n^p$

If p = n:

$N_{like} = n^n;$

In our case, n=4, hence:

$N_{like} = 4^4 = 2^8 = 256.$

(iii) The 24 Four-letter Anagrams of A, B, C, D.

01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
*A	A	A	A	A	A	B	B	B	B	B	B	C	C	C	C	C	C	D	D	D	D	D	D
*B	B	C	C	D	D	A	A	C	C	D	D	A	A	B	B	D	D	A	A	B	B	C	C
*C	D	B	D	B	C	C	D	A	D	A	C	B	D	A	D	A	B	B	C	A	C	A	B
*D	C	D	B	C	B	D	C	D	A	C	A	D	B	D	A	B	A	C	B	C	A	B	A

(iv) The number of distinct non-trivial machines N_S (X, Y) that can be synthesized with S internal, X input, and Y output states is (23):

$N_S(X, Y) = Y^{SX},$

In our case, X=Y=4.

For S=4:

$N_4 = 4^{4x4} = 2^{32} = 4{,}294{,}967{,}296.$

For S=24:

$N_{24} = 4^{24x4} = 2^{192}$, or about 6.3×10^{57}.

For S=256:

$N_{256} = 4^{256x4} = 2^{2048}$, or about 5×10^{616}.

REFERENCES

1. Curriculum Commission of the State Board of Education, State of California: Science Framework Field Review Draft. Science Subject Matter Committee, University of California, Berkeley, (May,1989).

2. Bateson,G.. "Metalogue: What is an Instinct?" in Steps to an Ecology of Mind.Balantine Books, New York, 38-60 (1972).

3. Ashby, R.: "The Brain of Yesterday and Today" in Mechanisms of Intellicience,Roger Conant (ed), Intersystems Publications, Seaside, 397-403 (1981).

4. Sacks, O.: "The Twins" in The Man Who Mistook His Wifw for a Hat, Harper and Row, New York,195-213 (1987).

5. Turing, A.: "On Computable Numbers, with an Application to the Entscheidungsproblem", Proc.London Mat.Soc.,2-/42:230-265 (1936).

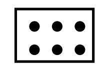
6. von Foerster, H.: "Kausalitaet, Unordnung, Selbstorganisation" in Grundprinzipien der Selbstorganisation, K. Kratky und F. Wallner (eds)t Wissenschaftliche Buchgesellschaft, Darmstadt (1990).

7. Eigen, M. und R.Winkler: Das Spiel: Naturgesetze steuern den Zufall. Piper, München (1975).

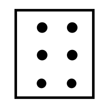
8. Laplace, P.S.: Essai Philosophique sur le Probabilités, Paris (1814).

9. Gillt A.: Introduction to the Theory of Finite State Machines, McGraw-Hill, New York (1962).

10 I was seeing a family when their 6 year old boy came home from school l/2 hour late and in tears: "I had to stay over in school"; "Why, what happened?"; "The teacher said I gave a fresh answer"; "What did you say?"; "She asked what is 3x2, and I said 2x3, and everybody laughed; then she put me in the corner". Now I interfered: "l think you gave a correct answer but can you prove it?" At once he drew on a piece of paper three columns with 2 dots each and said "That's 3x2:



Then he rotated the paper 90^{o} and said "That's 2x3.":



11. Von Foerster, H.: "Objects: Tokens for (Eigen-) Behavior" in Observing Systems, Intersystems Publications, Seaside, 273-286 (1984).

12. Buber, M.: Das Problem des Menschen, Lambert Schneider, Heidelberg (1961).

13. Gadel, K..' "Ueber formal unentscheidbare Saetze der Principia Mathematica und verwandter Systeme.' I Monatsh. Mat. Phys. 38: 173-193 (1931).

14. It came to my ears that significant steps for a proof of Goldbach's conjecture have been made. However, I could not confirm this rumor at the time of writing.

15. von Foerster, H.: "Wahrnehmen wahrnehmen" in Philosophien der neuen Technologien, Merve Verlag, Berlin, 27-38 (1989).

16. Searl, J.R., Churchland, P.M., Churchland, P.S.: "Artificial Intelligence: A Debate", Scientific American, 262: 25-39 (January, 1990).

17. Shorter, J.M.: "Other Minds, in The Encyclopedia of Philosophy, Macmillan, New York, 6: 7-13 (1967).

18. Johnson, G.: 'New Mind, No Clothes" The Sciences, New York Acad. of Sc., New York, 45-48 (August 1990).

19. Menger, K.: "Vorwort" in Einfuehrung in das Mathematische Denken. Friedrich Waismann, Gerold & Co., Wien, v-viii (1936).

20. This is a reference to Reference #10: The teacher apparently was not aware of the importance of the commutative law of multiplication, a consequence of the commutative law of addition. This elegant proof of commutativity in multiplication, using the invariance of area under rotation, could serve well as a tutorial device.

21. Ortega y Gasset, J.: Historia como sistema. Madrid (1941).

22. Translation M.v.F.
Freedom, flash from heaven,
Daughter from Elysium
Drunk with fire, heavenly
We enter your sanctuary.

23. von Foerster, H.: "Molecular Ethology, An Immodest Proposal for Semantic Clarification" in Molecular Mechanisms in Memory and Learning, Georges Ungar (ed), Plenum Press, New York, 213-248 (1970).